The separate role of pairing amplitude and phase in cuprates.

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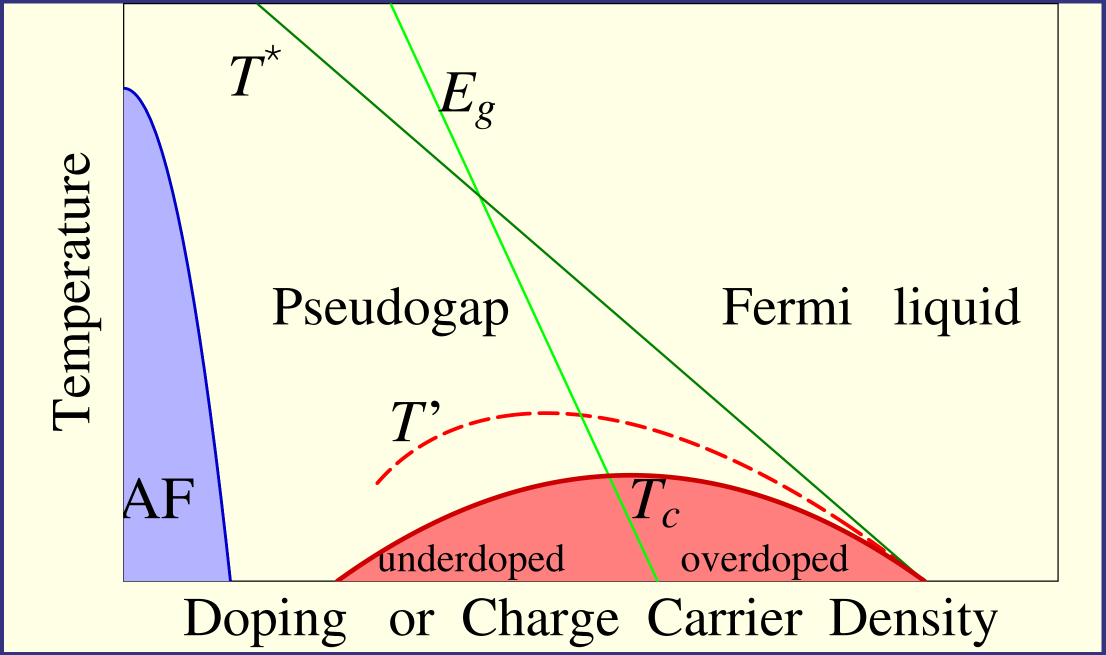
outline

- What is the real phase diagram of cuprates?
- Why are phase and amplitude relevant?
- How is the pseudogap related to pairing and phase fluctuations?

phase diagram of cuprates

pseudogap: two or four regimes?

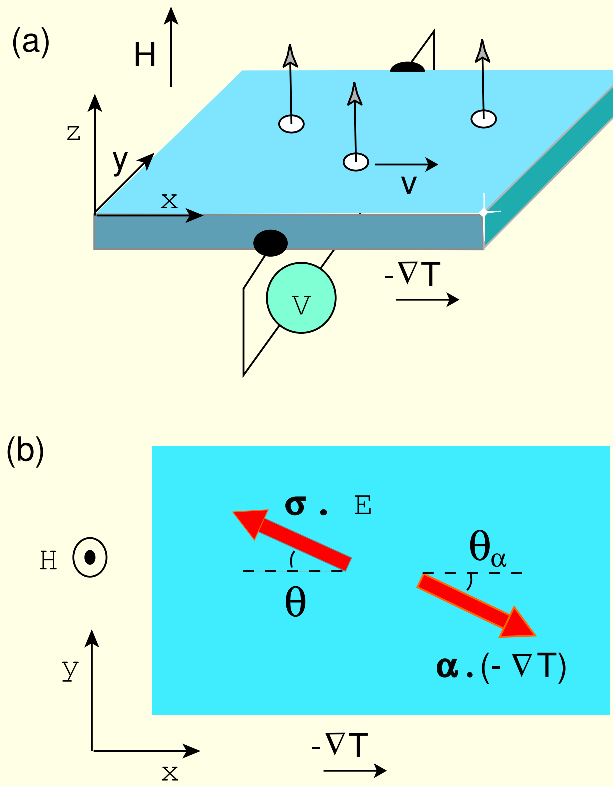


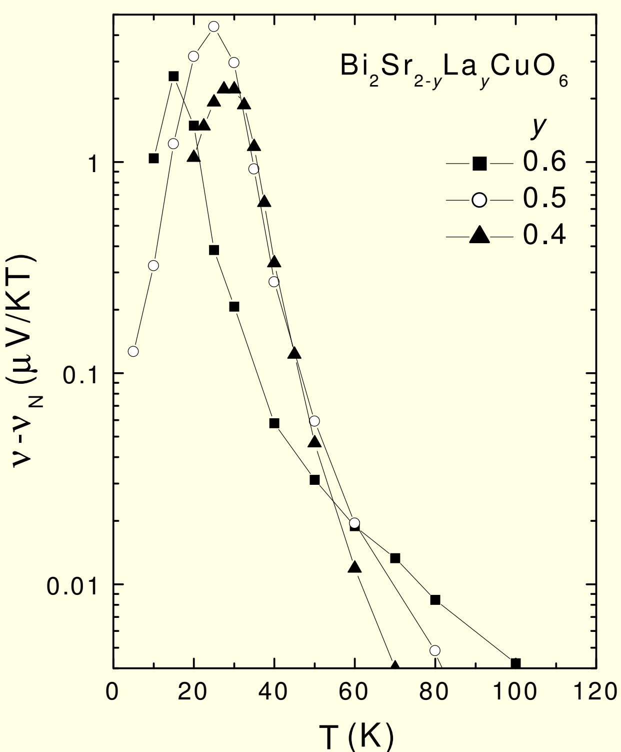
T': Nernst effect, Hall effect, specific heat.

E_g: specific heat, spin susceptibility, ARPES,...

Nernst effect: experiment

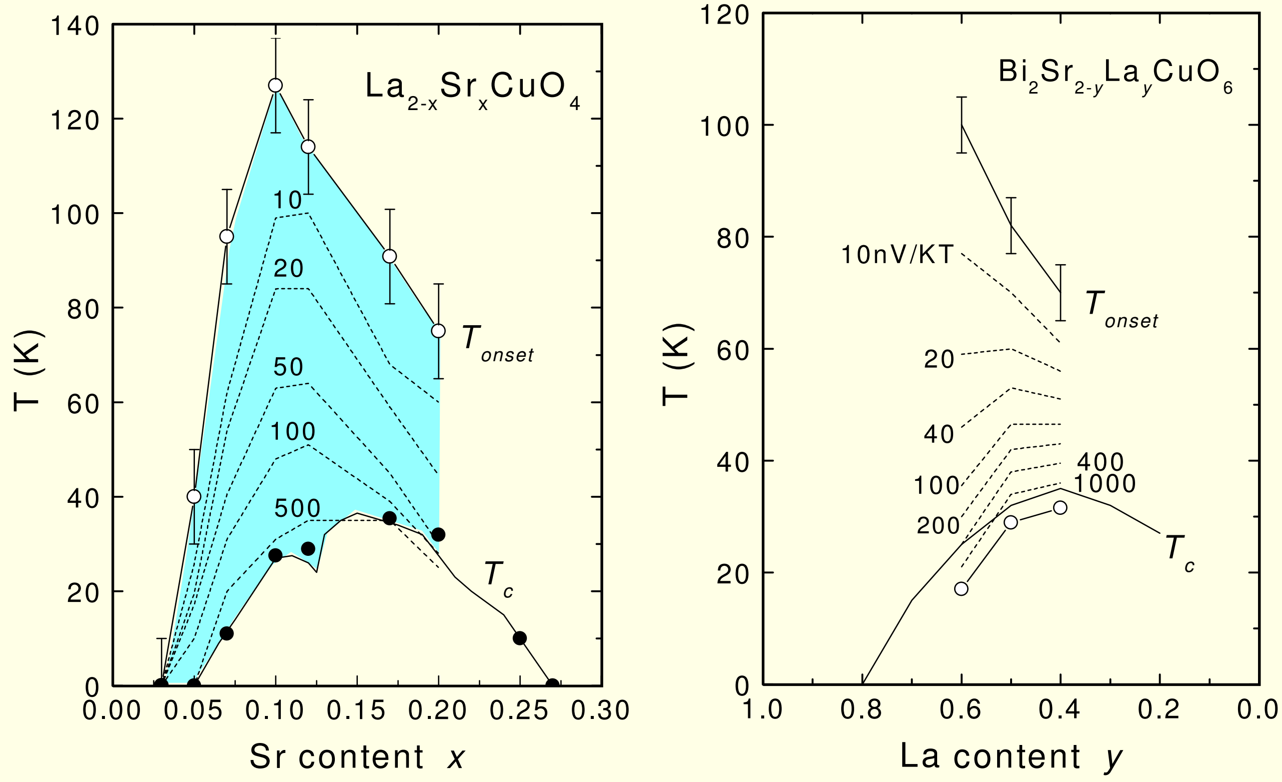
$$\nu = \frac{E_y}{|\partial_x T| B}$$





ref: Wang *et al*, cond-mat/0108242
Wang *et al*, PRL 88, 257003 (2002)

Nernst effect: phase diagram



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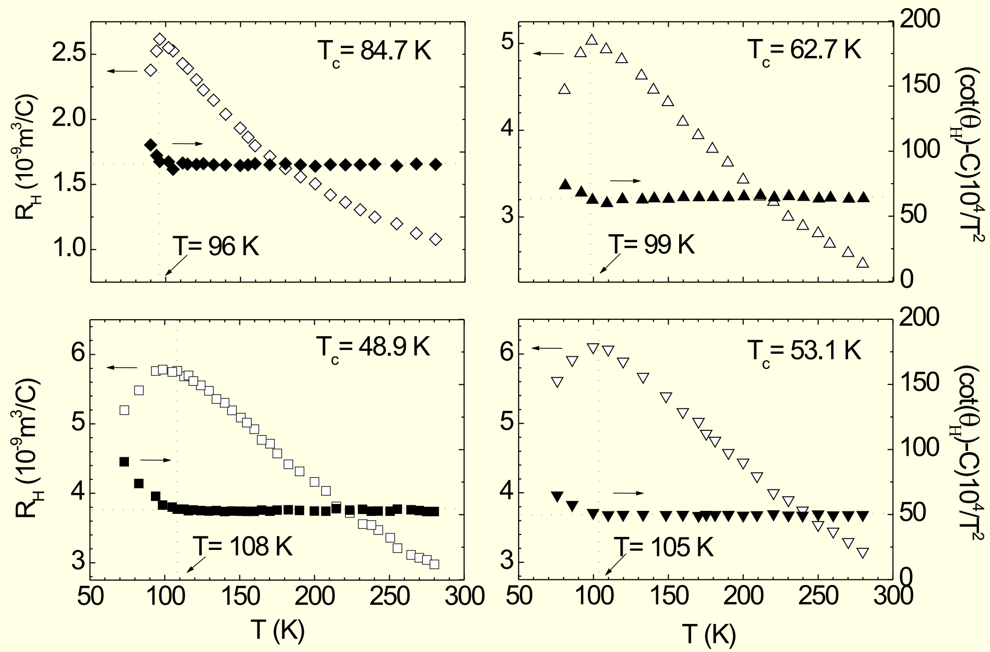
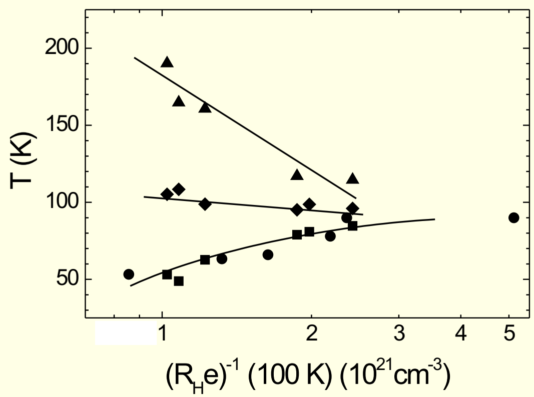
Hall effect



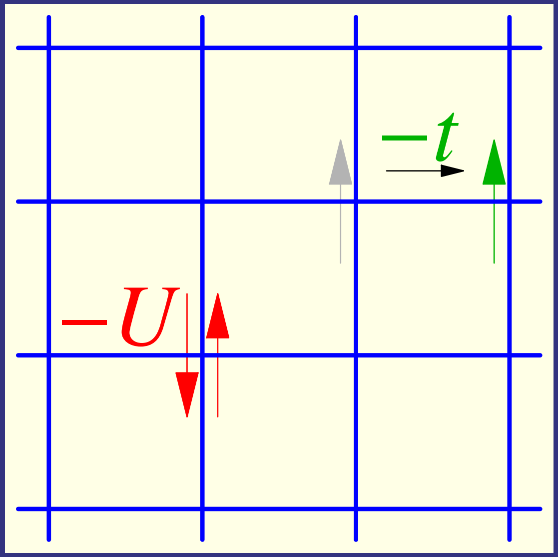


ref: Matthey *et al*, PRB 64, 24513 (2001)

model

Experiments show d-wave (s-wave) pairing
$\Rightarrow$ attractive d-wave Hubbard model:

$$H = -t \sum_{\langle i,j \rangle \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} - U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$



introducing phase and amplitude

Hubbard-Stratonovich transformation:

$$e^{-Un_{i\uparrow}n_{i\downarrow}} = cst \int d\psi d\psi^* e^{-\frac{|\psi|^2}{U} + \psi n_{i\uparrow} + \psi^* n_{i\downarrow}}$$

$\psi \longrightarrow |\psi| e^{i\phi}$

- Universality class of superconductors is 2DXY or 3DXY $\Rightarrow$ phases are relevant!
- The amplitude $|\psi|$ is the energy scale. ($|\psi|_{T=0} = \Delta_0$).

effective action

Integrate fermionic operators $\Rightarrow$ standard action:

$$S[\psi] = S_0(|\psi|) + S_\phi(\vec{\nabla}\psi)$$

$S_0 = F_{BCS}$ for a fixed $|\psi|$.
$S_\phi = \int dV \, |\nabla\psi|^2$ is the phase part.
The free energy is

$$F(T) = -\frac{1}{\beta} \log \int d\psi_1 \cdots d\psi_N \; e^{-\beta S[\psi]}$$

Two parameters: T_0 and V_0.

goal and method

What is the influence of the pairing field ψ on thermodynamics?

Two methods

- variational: the free energy is minimal with respect to $|\psi|$. (amplitude correlations are neglected)
- average value: expand the energy around the average amplitude $\langle|\psi|\rangle$.

variational method

Polar coordinates:

$$\int d^2\psi = \int_0^\infty d|\psi||\psi| \int_0^{2\pi} d\phi$$

Fix the amplitude to a constant $|\bar{\psi}|$. The free energy is:

$$F = -\frac{1}{\beta} \log \int_0^{2\pi} D\phi \; e^{-\beta\left(S_0(\bar{\psi}) - \log(|\bar{\psi}|)V/\beta + S_\phi\right)}$$

equation of the variational method

Minimize:

$$\frac{\partial F}{\partial |\bar{\psi}|} = 0$$

$\Rightarrow$

$$\underbrace{\frac{\partial S_0(|\bar{\psi}|)}{\partial |\bar{\psi}|}}_{\text{amplitude part}} - \underbrace{\frac{1}{\beta |\bar{\psi}|}}_{\text{Jacobian term}} + \underbrace{c|\bar{\psi}| \ \langle S_\phi \rangle_{S_\phi}}_{\text{phase fluctuations}} = 0.$$

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$\Downarrow (static)$

BCS

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$\Downarrow (static)$ $\qquad\qquad$ $\Downarrow (|\psi| = cst)$

BCS $\qquad\qquad$ XY model

solving the variational equation

$$\frac{\partial S_0(|\bar{\psi}|)}{\partial|\bar{\psi}|} - \frac{1}{\beta|\bar{\psi}|} + c|\bar{\psi}| \ \langle S_\phi \rangle_{S_\phi} = 0.$$

solving the variational equation

$$\frac{\partial S_0(|\bar{\psi}|)}{\partial|\bar{\psi}|} - \frac{1}{\beta|\bar{\psi}|} + c|\bar{\psi}| \ \langle S_\phi \rangle_{S_\phi} = 0.$$

$$\frac{|\bar{\psi}|^2}{U} - \frac{|\bar{\psi}|^2}{W} \int_{-\mu}^{W-\mu} d\xi \, \frac{\tanh\left[\sqrt{\xi^2 + |\bar{\psi}|^2}/(2T)\right]}{\sqrt{\xi^2 + |\bar{\psi}|^2}} - \frac{\bar{\psi}_0^2}{\beta} + |\bar{\psi}|^2 \ \left\langle |\vec{\nabla} e^{i\phi}|^2 \right\rangle_{S_\phi} = 0.$$

solving the variational equation

$$\frac{\partial S_0(|\bar{\psi}|)}{\partial|\bar{\psi}|} - \frac{1}{\beta|\bar{\psi}|} + c|\bar{\psi}| \; \langle S_\phi \rangle_{S_\phi} = 0.$$

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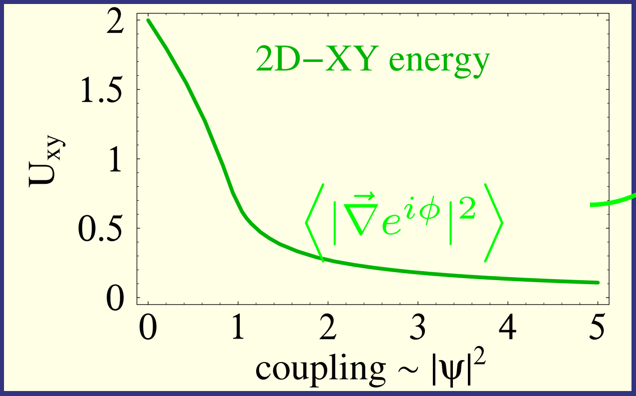
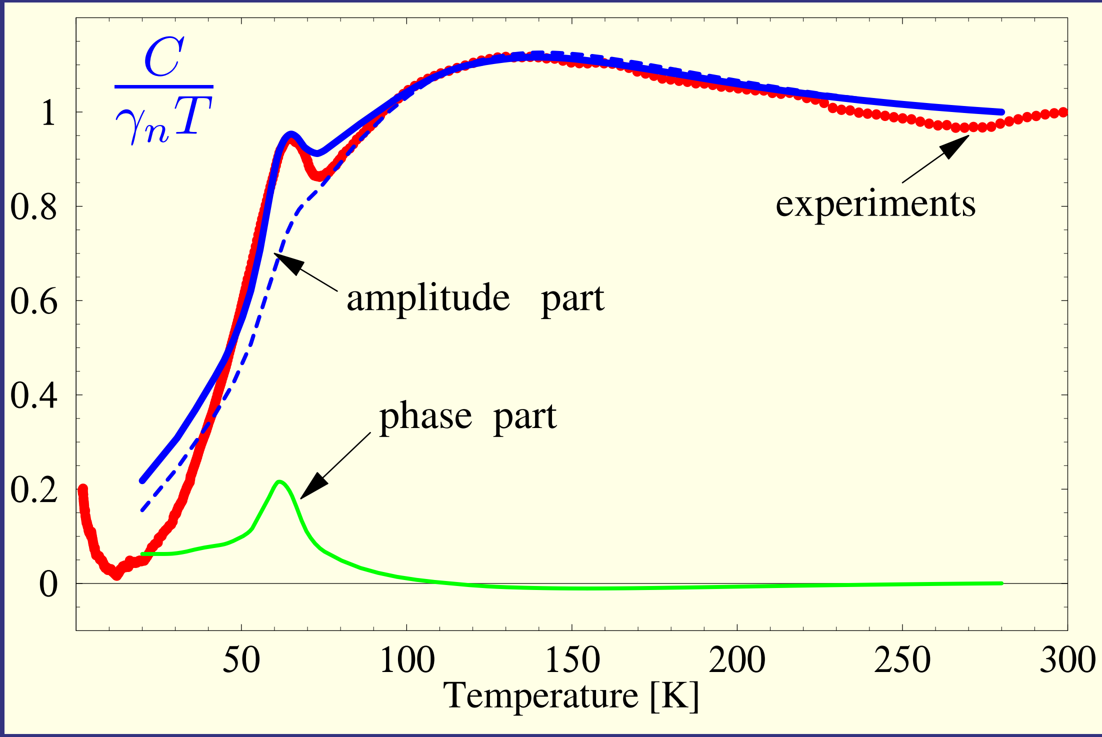


variational method: specific heat

The specific heat is $C = C_{amplitude} + C_{phase}$



$YBa_2Cu_3O_{6.73}$ specific heat. (exp.: Loram *et al*, PRL 71, 1740 (1993))

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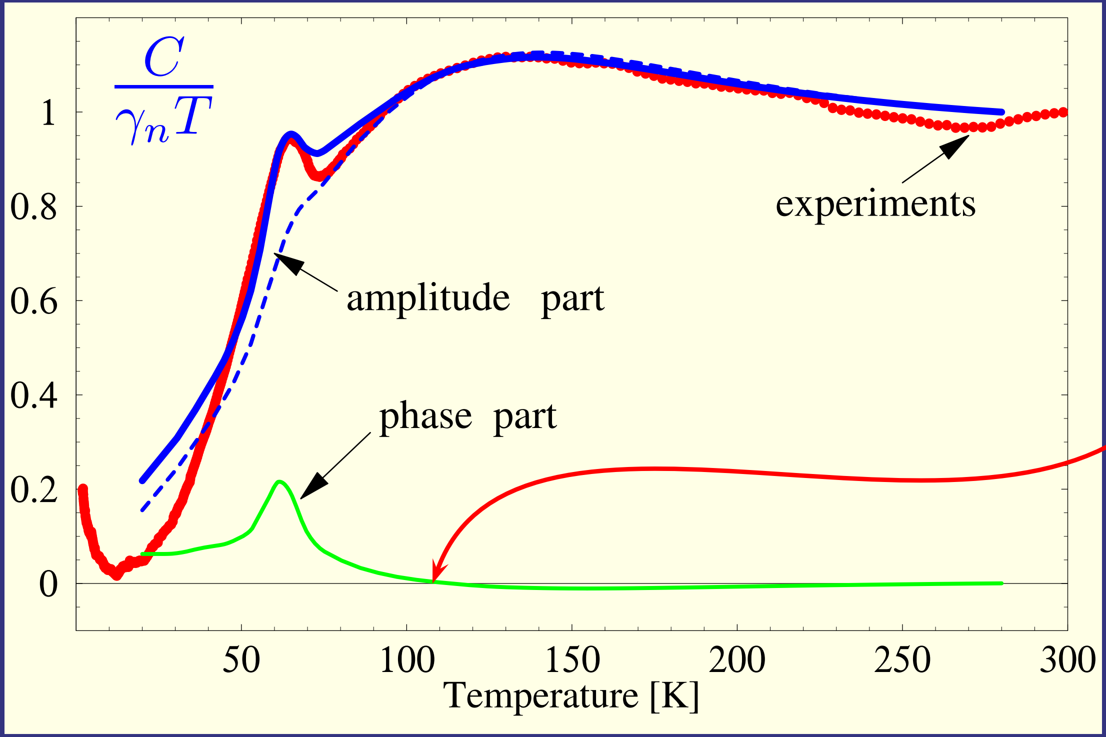

T_ϕ is the temperature above which phase correlations are negligible.

$YBa_2Cu_3O_{6.73}$ specific heat. (exp.: Loram *et al*, PRL 71, 1740 (1993))

average value approach

- Expand the energy around the average amplitude:

$$U = \langle S \rangle_S \approx S_0(\langle |\psi| \rangle) + \langle S_\phi \rangle_S + \mathcal{O}(\langle \delta |\psi| \rangle^2)$$

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$$U = \langle S \rangle_S \approx S_0(\langle |\psi| \rangle) + \langle S_\phi \rangle_S + \mathcal{O}(\langle \delta |\psi| \rangle^2)$$

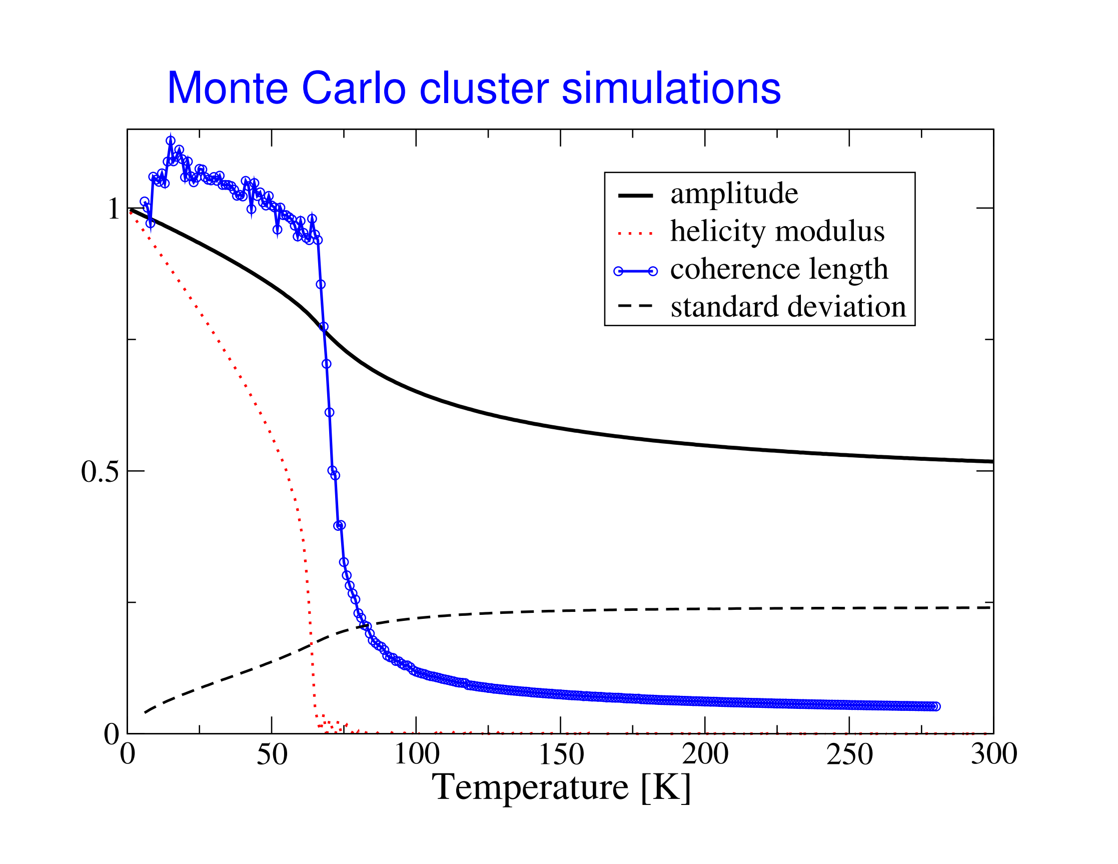
- Replace averages over S by averages over the Ginzburg-Landau action $S_{GL} = U_{GL}(|\psi|) + S_\phi$:

$$\langle \cdots \rangle_S \quad \longrightarrow \quad \langle \cdots \rangle_{S_{GL}}$$

Computation

$Z = \int D\psi \; e^{-\frac{V_0}{T}\sum_i(\sigma(t|\psi_i|^2+\frac{1}{2}|\psi_i|^4)+\frac{1}{2}|\nabla\psi_i|^2)}$ where $t = T/T_0 - 1$, and

$S_\phi = \sum_i \frac{1}{2}|\nabla\psi_i|^2$



ouput needed:

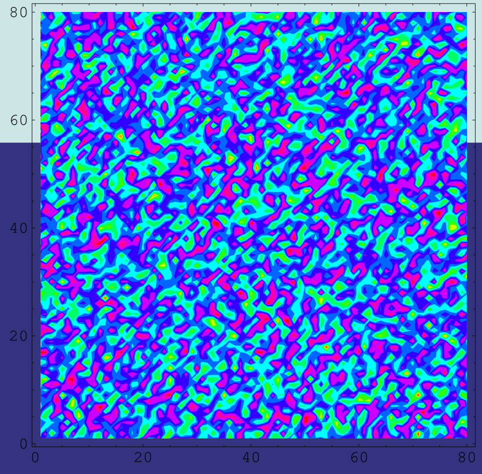
$\langle|\psi|\rangle$

$U_\phi = \langle S_\phi \rangle$

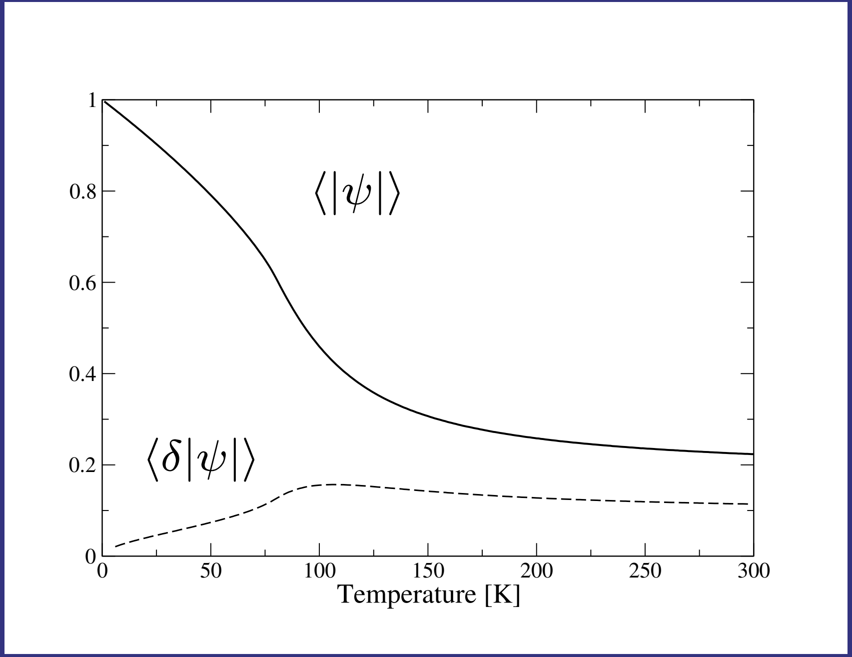
for <u>all</u> parameters.

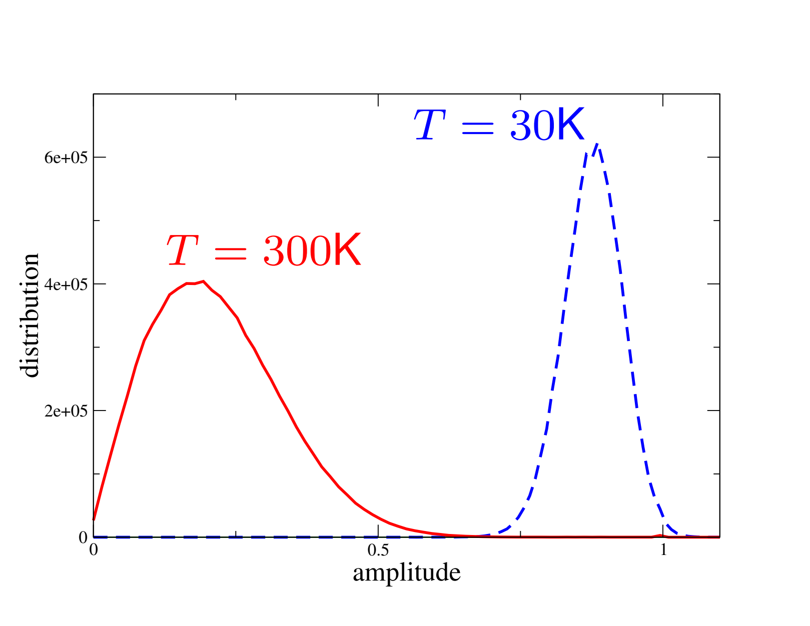
Fluctuations

$$\langle \delta|\psi| \rangle = \sqrt{\langle |\psi|^2 \rangle - \langle |\psi| \rangle^2}$$



Amplitude Distribution

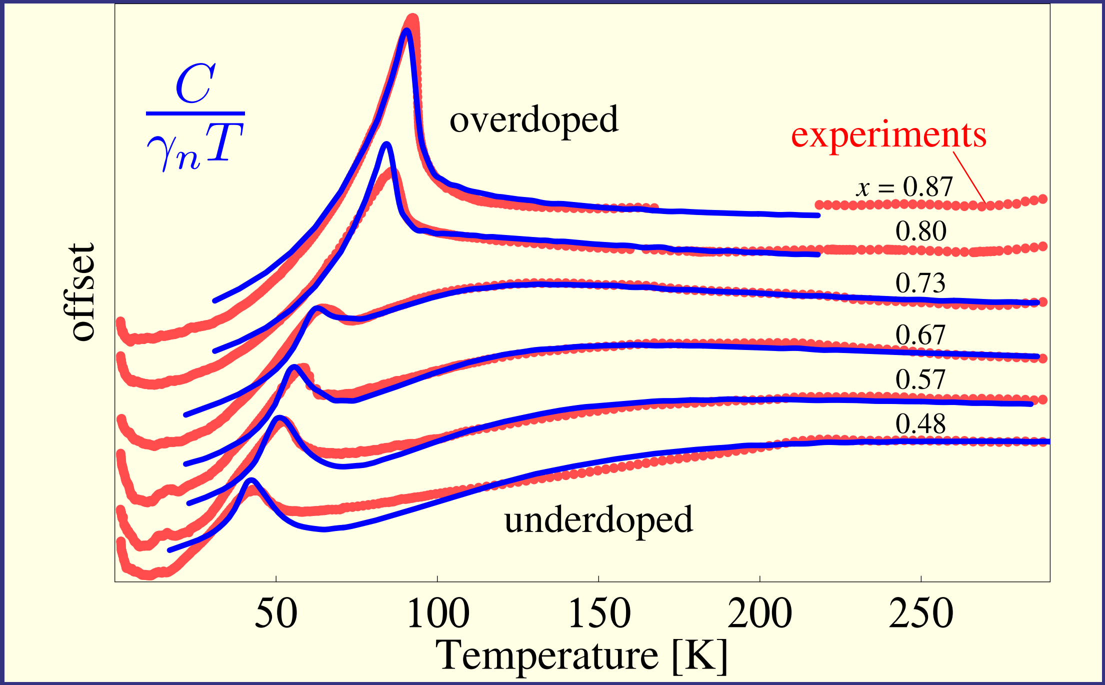




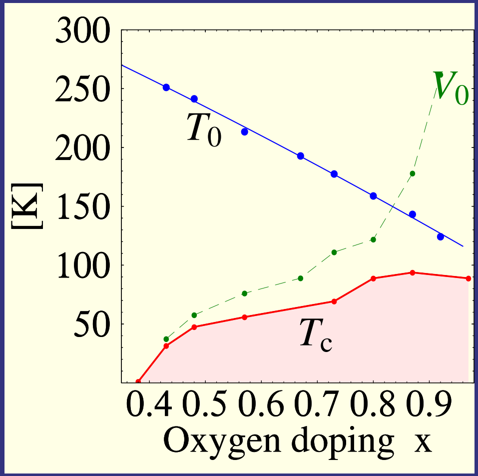
average value: YBCO specific heat

The specific heat is $C = C_0\left(\langle|\psi|\rangle\right) + C_{phase}$

where $\langle|\psi|\rangle = Z^{-1}\ \int D^2\psi\ |\psi|\ e^{-\beta S_{GL}[\psi]}$



V_0 (phase stiffness) and T_0 (MF temperature) are extracted from the best plots: underdoped, overdoped

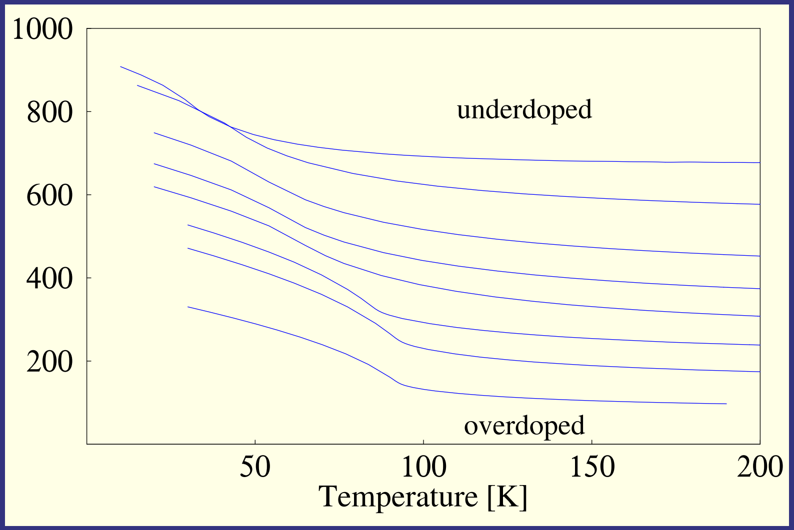


$YBa_2Cu_3O_{6+x}$: underdoped: 2D d-wave, overdoped: 3D d-wave (s-wave)

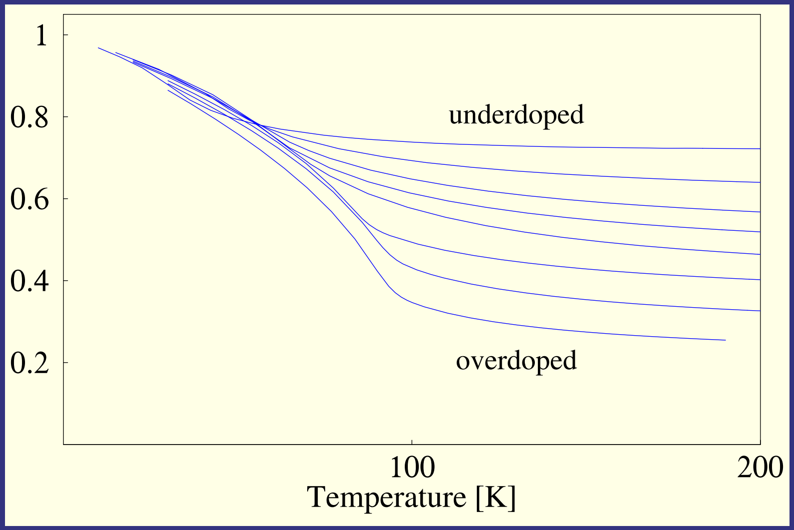
average value: amplitude for YBCO

The average amplitude is $\langle|\psi|\rangle = Z^{-1} \int D^2\psi \, |\psi| \, e^{-\beta S_{GL}[\psi]}$

$\langle|\psi|\rangle$



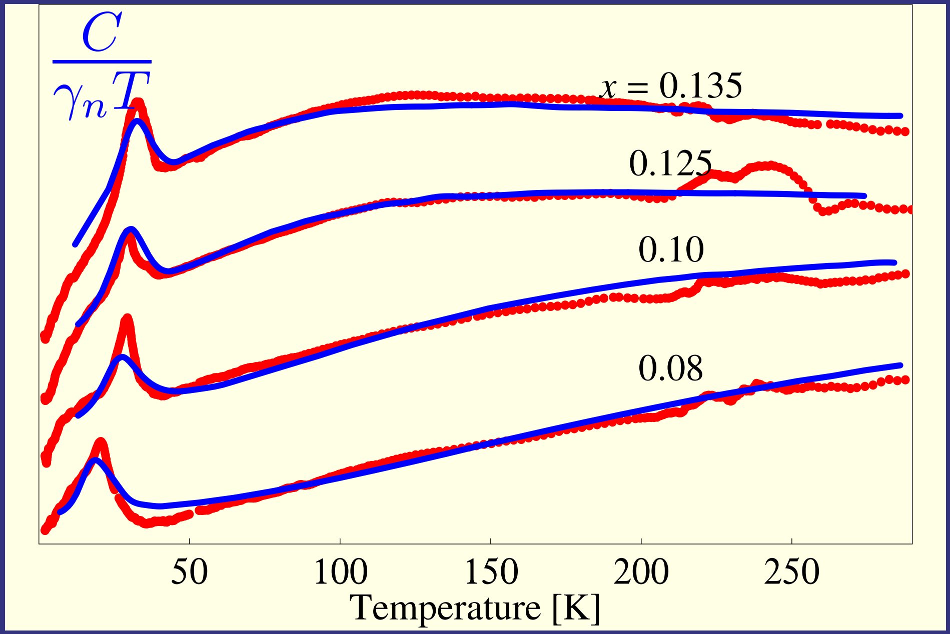
$\langle|\psi|\rangle/\psi_0$



average value: LSCO specific heat

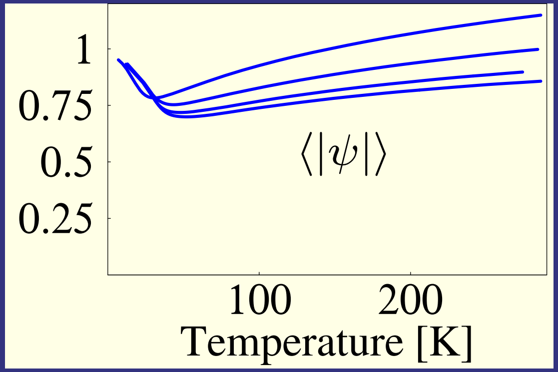
$$C = C_0\left(\langle|\psi|\rangle\right) + C_\phi$$

x	0.08	0.1	0.125	0.135
V_0	30	45	50	54
T_0	490	359	224	192



$La_{2-x}Sr_xCuO_4$ specific heat for different dopings.

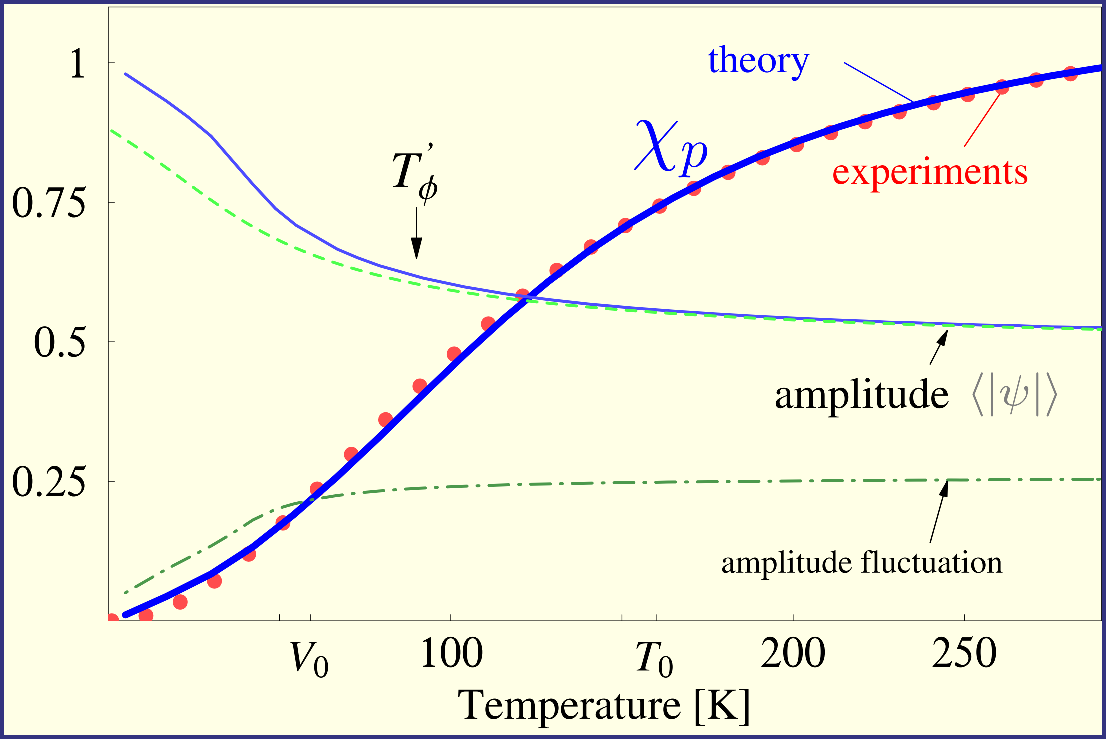
2D d-wave (exp.: Loram *et al*)



average amplitude.

magnetic spin susceptiblity χ_p

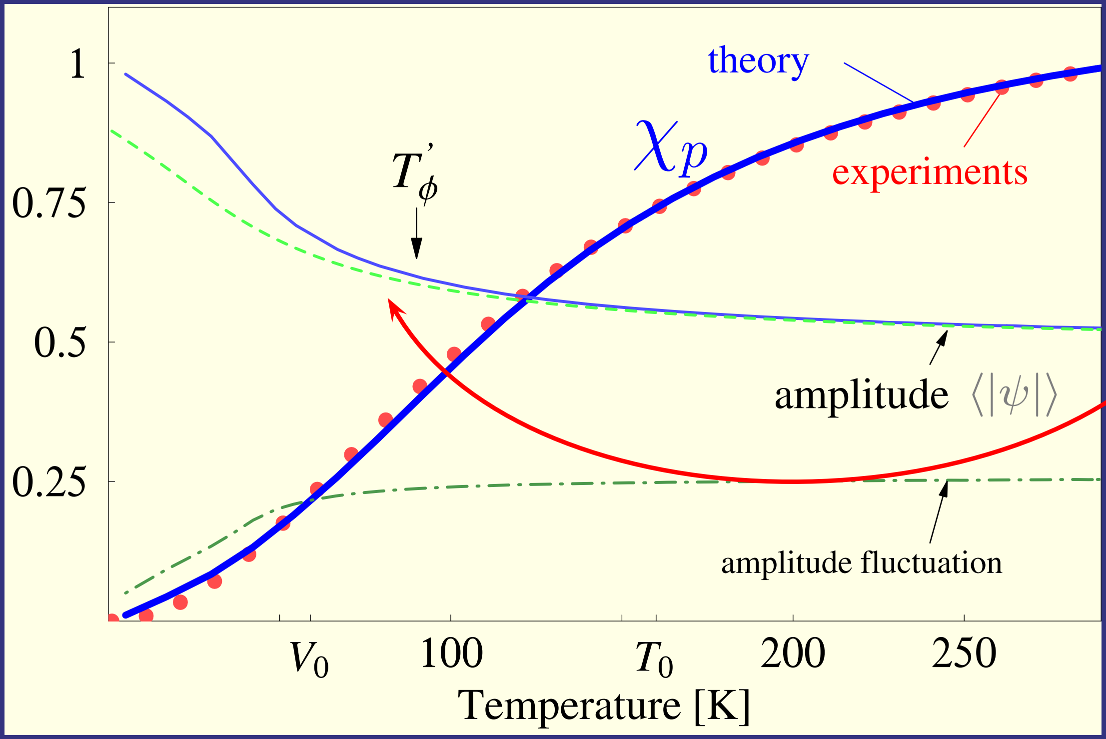
$\chi_p = \chi_{BCS}\left(\langle|\psi|\rangle\right)$ where $\langle|\psi|\rangle = Z^{-1} \int D^2\psi\ |\psi|\ e^{-\beta S_{GL}[\psi]}$



$YBa_2Cu_3O_{6.63}$ magnetic spin susceptibility: d-wave. (exp.: Takigawa *et al*)

magnetic spin susceptiblity χ_p

$\chi_p = \chi_{BCS}\left(\langle|\psi|\rangle\right)$ where $\langle|\psi|\rangle = Z^{-1} \int D^2\psi\; |\psi|\; e^{-\beta S_{GL}[\psi]}$

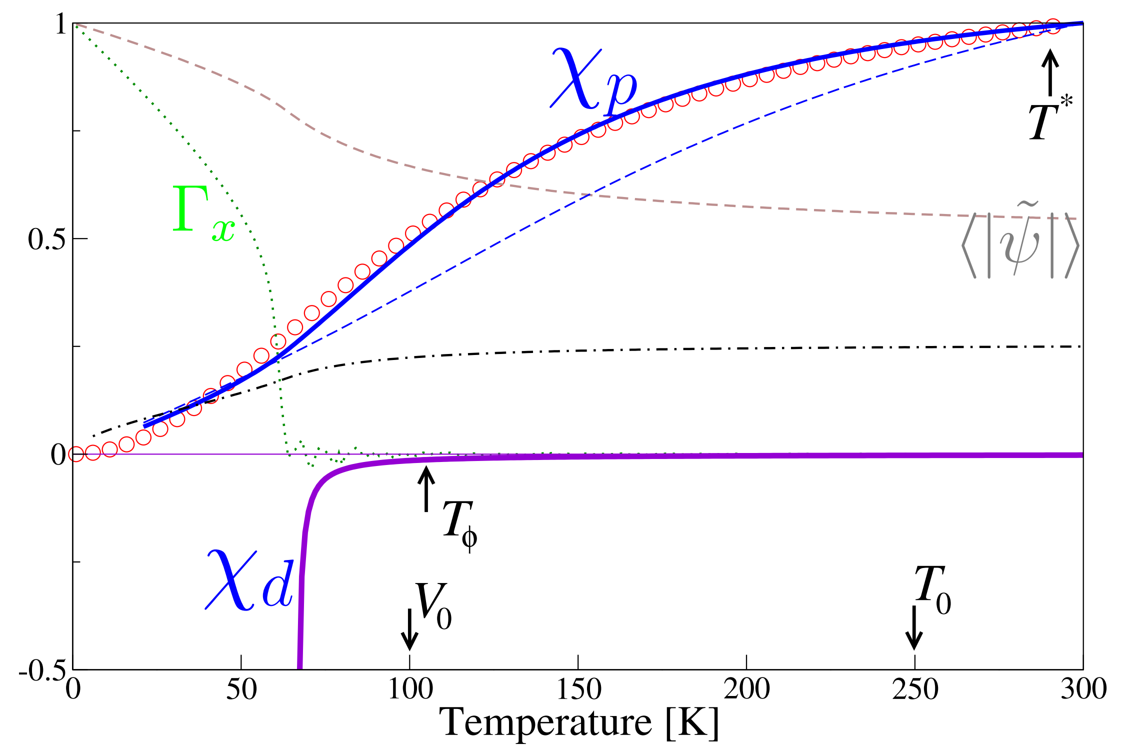


T'_ϕ is the temperature above which phase does not influence the amplitude (and χ_p)

$YBa_2Cu_3O_{6.63}$ magnetic spin susceptibility: d-wave. (exp.: Takigawa *et al*)

magnetic susceptiblity $\chi = \chi_p + \chi_d$

χ_p is the paramagnetic spin susceptibility and χ_d is the diamagnetic susceptibility.

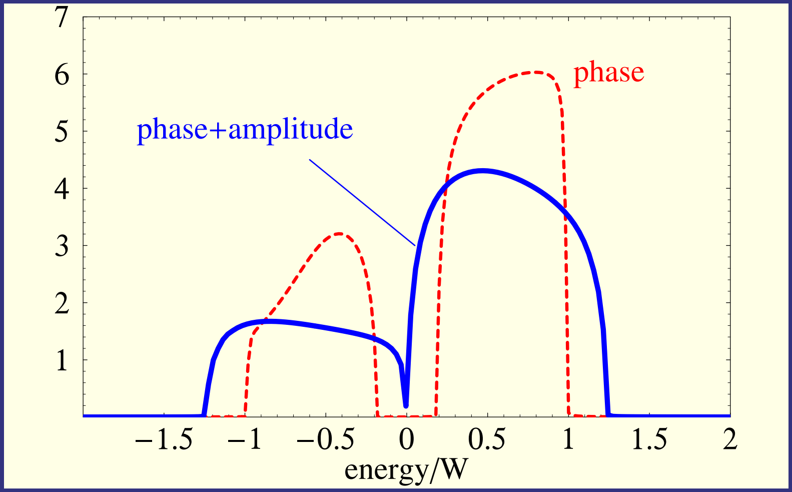


YBCO: magnetic spin susceptibility and diamagnetic susceptiblity.

density of states: CPA

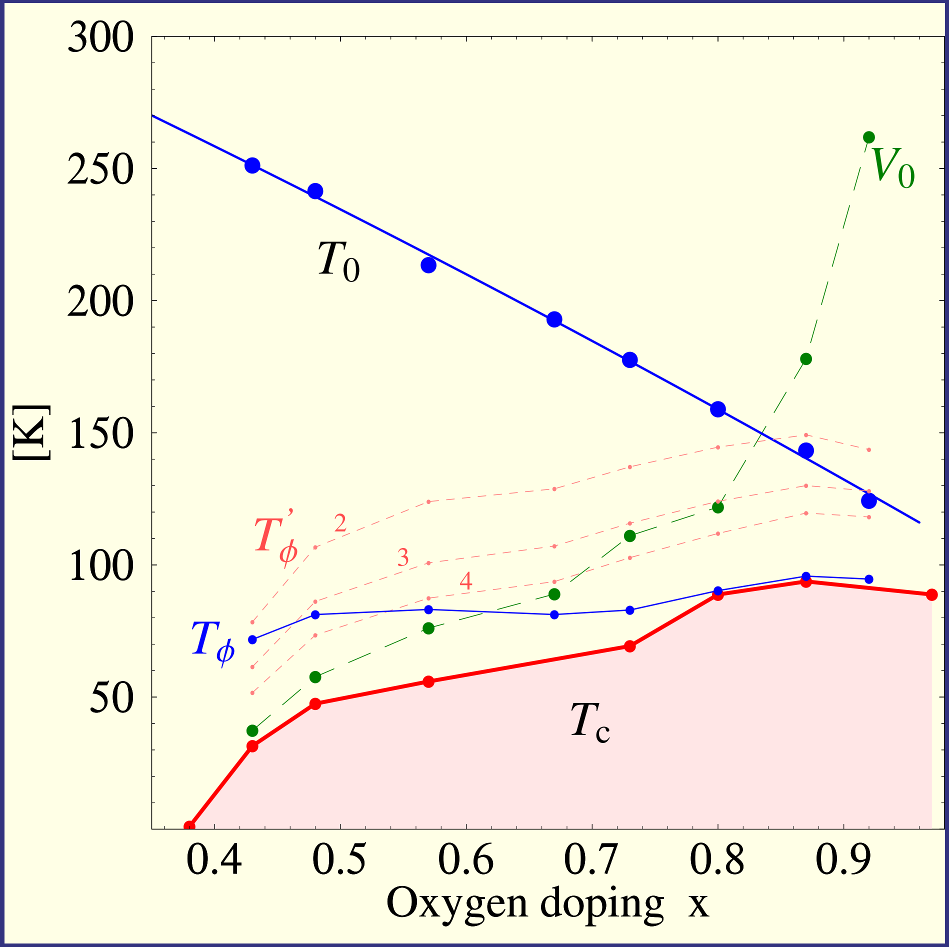
CPA (Coherent Phase Approximation):

$$\underbrace{\langle G(r,r)\rangle_{\psi_r}}_{\text{average local GF}} = \underbrace{\mathcal{G}(0,z)}_{\text{effective local GF}}$$

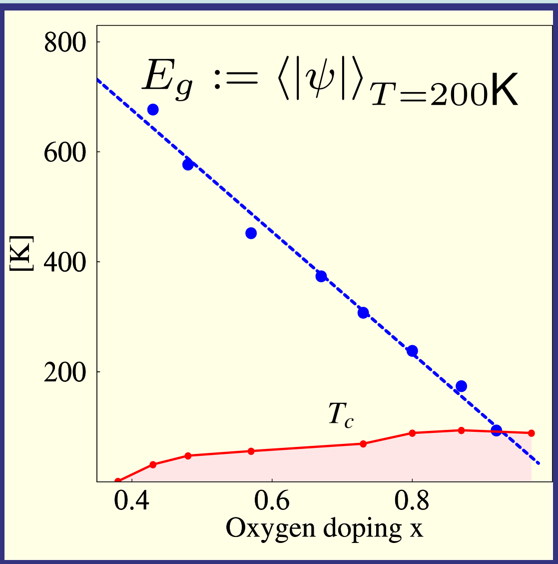


Average over amplitude and phase (thick line).

resulting phase diagram.



Phase diagram of $YBa_2Cu_3O_{6+x}$.



pseudogap energy scale E_g

$\Rightarrow T_0 \sim T^*$

$\Rightarrow T_\phi \sim T'_{Hall}, T'_{\text{Nernst}}$.

conclusion and perspective

- Superconductivity and pseudogap have the same origin.
- Two regimes in the pseudogap: correlated (phase) and uncorrelated (amplitude).
  ($\neq$ phase scenario.)

To do: Nernst effect, transport, etc..